*International Journal of Research in Advent Technology, Vol.4, No.5, May 2016*
*E-ISSN: 2321-9637*
*Available online at www.ijrat.org*

# Robust Analysis of Linear Time Invariant systems using dynamic state observer

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**Abstract-**In this paper, we investigate performance robustness, error dynamics, and the effect of measurement as well as state disturbance on different linear time-invariant systems which works in different environment, using the dynamic state observers. These observers are commonly used in control engineering, but now they were also used in modern automotive powertrain application. Their approach is homologous to Kalman Filters except the consideration that perturbations are deterministic, not stochastic. These observers exhibits reliability and efficiency in estimation of states of the system. In this analysis, we use the application of observer theory on different linear time-invariant models. And investigate the effects of state and measurement disturbances that take place during the normal operation of the system, and analyze the performance robustness.

**Index Terms-** Dynamic state observers, Control theory, Linear Time Invariant system, State disturbance, Measurement Disturbance, Observer theory**.**

## 1. INTRODUCTION

There has been tremendous work going on study of dynamic state observers and there results were well used in control engineering. The concept of the dynamic state observer or commonly called as observer theory was first introduced by Luenberger in early 70's [1]. He has given the complete study of observers which are derived in both continuous time as well as in discrete time. Ever since, the dynamic state observers transformed the control theory onto another level, by approximation of the state vector using direct measurements of the systems output which is available to us. In some of the systems, by direct measurements all the states variables can't be easily observed. In that case, the expensive measuring devises might be needed to measure states. In addition to that, some systems state variables may inaccessible for measuring directly. For these difficulties, state observer gives the solution by approximation of system states for which information may or may not be available. This makes state observers used as model based method in state feedback control system. From last four decades dynamic state observers and its methodology were used in various fields of engineering.

The recent field that benefitted by dynamic state observers is industrial used linear time-invariant system. Specifically, we discuss some of the linear time-invariant systems which were used in different field like internal combustion engine in automotive powertrain, BLDC motor which is extensively used in home appliances, medical equipment's, and level control system mainly used in chemical and petroleum industries. These systems are used extensively in many application. This make us to improve their efficiency and reliability.
Internal combustion engines, we know that if modern internal combustion engines need to be used as primary propulsion source, they have to decrease their emission and improve its fuel efficiency as per government mandate. Which results in development of new technologies such as advanced idle speed control scheme[2], engine start-stop etc. the more widely used method to solve the efficiency problems is to utilize the knowledge and effectiveness of model based method, uses the dynamic state observers.

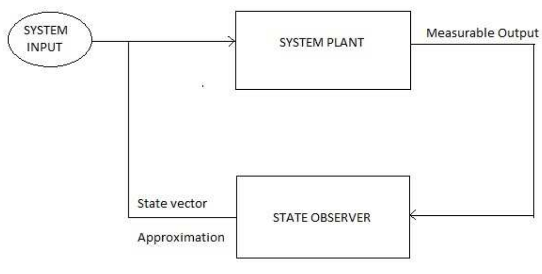


Fig.1 State observer feedback flow diagram.

In mid-90's, Hrovate introduced the model based controllers to achieve fuel efficiency and better speed control in automotive industries [3]. At the same time optimal control is used in robust controller that represents the influence of unmolded dynamics like measurement noise, disturbances, model inaccuracies, transient loads of internal combustion system. Therefore for better performance of the LTI systems control engineering committee has developed many

Other methods like Kalman controllers [4-5], adaptive controllers [6-7], and non-linear adaptive fuzzy logic [8-10]. Kalman filters are used widely because of its abilities to deal with noise and disturbance. These filters are very well applied to many control engineering problems and found success. But, it has its limitations regarding assumption of noise has Gaussian disturbance. Which does not hold well in real time applications. And also its implementation was computationally very complex. Adaptive control methods also has its limitations as, it is been limited by their non-linearity and need for highly reliable models run's in multiple frequency ranges.

The solution for all this problems is the robust, lower order, dynamic state observer model based method presented in this paper. This model is efficient enough to use in real time LTI systems. It minimizes the computational time or delays. As though we know the observer theory in literatures, the observer performance has not been investigated completely under non-ideal conditions and disturbance. In this paper, we mathematically model the different linear time invariant systems, which is represented in state space using real time parameters of respective models. And discuss the variations in state space model of linear time invariant systems, due to disturbances included in the system. Sensors measurement disturbance and state disturbance are investigated by analyzing their effects on the dynamic state observer performance and observer error dynamics. Observer model designed in order to estimate the states of the system will be discussed in next section 2. And the mathematical model linear time invariant systems is discussed in section 3. The expected result and analysis of robustness of the systems are discussed in section 4 and conclusion in the end.

## 2. OBSERVER DESIGN

[1]State Observer is a system which approximate the internal states of the real system which is necessary for control. To design the feedback control system, it is convenient to assume that the system states can be available through measurement of input and output of the system which are to be controlled. These observers were the basis for many real time applications.

In this section, we discuss the complete derivation state observer design and its error dynamics. Generally state observer is designed to reconstruct the state of the linear system [11]. The state space representation of the linear time invariant system is as below.

$$\dot{x}(t) = Ax(t) + Bu(t) + w$$

$$y(t) = Cx(t) + Du(t) + v$$

Where, t=time; p= number of states in the state vector; q=number of states in the output vector; r=number of inputs to the system; A=state transition matrix; B= input matrix; C=output matrix, x(t) = state vector of the plant with dimension p x p; y(t)= output vector of the plant with dimension q x p; u(t)= input vector of the plant with dimension r x 1; v= measurement disturbance; w=state disturbance.

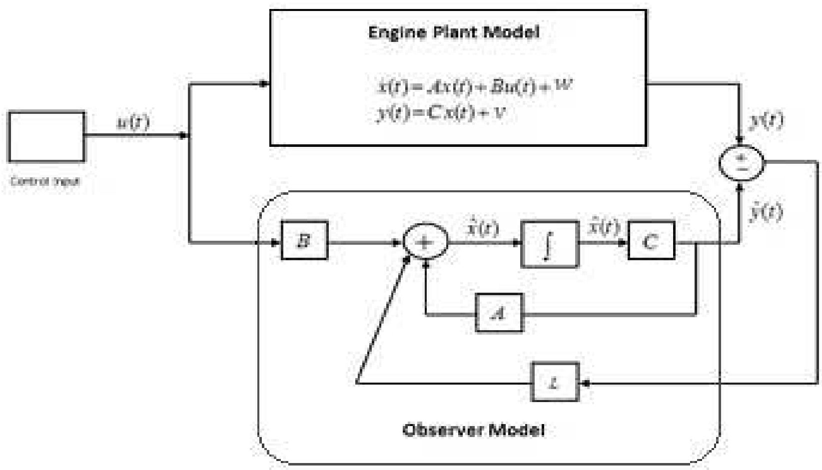


Fig.2 Block diagram of state observer system

This form of state space representation includes the effect of the state disturbance (W), measurement disturbance or output noise (V). This state space model is similar to the Kalman filter. In this work, effect of these disturbances on the system is our interest which is represented in the model. The state space model mathematically proves us that changes in the state vector is affected by its values at time t sec, disturbances on the state vector, and the system input. The output of the system is measured from state vector taking the errors of the measuring devices into consideration.

[11]The state observer model is presented in the form similar to the plants state space model as shown below.

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L(y(t) - \hat{y}(t))$$

$$\hat{y}(t) = C\hat{x}(t) + Du(t)$$

Where $\hat{x}(t)$ denotes the estimated state vector, $\hat{y}(t)$ denotes the estimated output of the observer model. By these equations we can see that observer state space equation is differs from plant state equation. Because of the observer gain L is used as factor of correction for output error$(y(t) - \hat{y}(t))$. If this output error is large then by using the observer gain L, we can reduce the error. This correction may result in state observer error to zero. If this output error is less then both the observer state model and plant model are identical.

There are so many methods to determine the observer gain, mostly used methods are Pole placement method [11], and Linear Quadratic Regulator method [12]. The state observer performance is analyzed by the help of the observer error dynamics. The error converges to zero when the observer is well designed.

The state observer error dynamics derived as

$$e(t) = x(t) - \hat{x}(t)$$

$$\dot{e}(t) = \dot{x}(t) - \dot{\hat{x}}(t)$$

$$\dot{e}(t) = (Ax(t) + Bu(t) + W) - (A\hat{x}(t) + Bu(t) + L(Cx(t) + V - C\hat{x}(t))$$

$$\dot{e}(t) = (A - LC)(x(t) - \hat{x}(t)) + W - LV$$

$$\dot{e}(t) = (A - LC)e(t) + W - LV$$

This is the equation for state observer error dynamics. Our objective is to analyze the effects of state and measurement disturbance that is W and V respectively, on observer error dynamics and observer's overall performance.

## 3. MATHEMATICAL MODELLING OF LINEAR TIME INVARIANT SYSTEMS.

### *3.1 Internal combustion engine.*

The internal combustion engine mathematical model is presented by C.E Baumgartner, H.P Geering, C.H Onder, E. shafal [13]. This model is firstly implemented for ideal speed control of the internal combustion engine (dodge neon) and it has three inputs and three states. Later it is modified by Omekanda and Zohdy which is published in 2014 [14]. Where its inputs are reduced to one and used to design reduced order observer, to make our work easy and simpler. The latter model similar to this is used for the analysis of the state observer dynamics with state vector x(t) = [x1, x2, x3] and input u(t). This model is modified with the throttle angle of 7 deg, commands a speed of 680 RPM. The state space model of internal combustion engine is given as follows.

$$\dot{x}(t) = \begin{pmatrix} -35 & 0 & 0 \\ 4.212 & -3.822 & -6.552 \\ 0 & 3.2 & 1.84 \end{pmatrix} x(t) + \begin{pmatrix} 35 \\ 0 \\ 0 \end{pmatrix} u(t)$$

$$y(t) = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} x(t)$$

u(t) = commanded throttle angle in degrees,
x1 = throttle angle in degrees,
x2 = intake manifold pressure in torr,
x3 = speed of the engine in RPM.

### *2.2 Mathematical model of BLDC motor*

A dc motor equivalent circuit is illustrated [15] as shown in the circuit diagram shown below

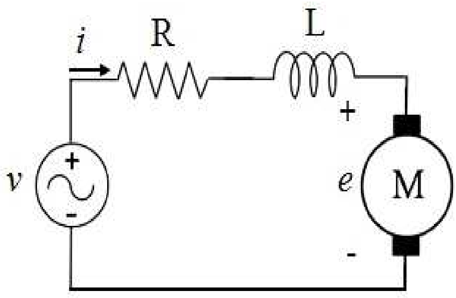


Fig.3 DC Motor electrical equivalent circuit.

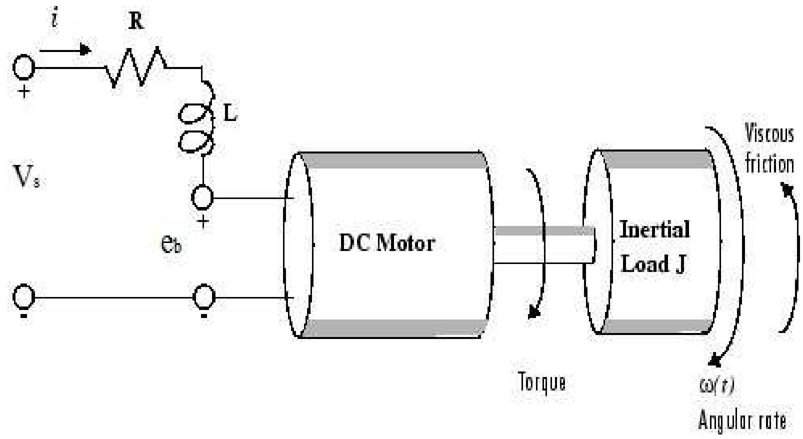


Fig.4 Typical dc motor electro-mechanical equivalent circuit.

Form the above Fig.3 and Fig.4 we will form the equations, which can describe the relationship of operation. The basic components of the dc motor are represented as armature resistance (R), armature inductance (L), and the back emf ($e_b$).
Using the Kirchhoff's Voltage law, KVL, the following equation is obtained

$$v_s = Ri + L\frac{di}{dt} + e_b \quad (3.1)$$

Where $v_s = source\ voltage\,(DC);$
$i$ = Armature current;

At study state we know that,

$$v_s = Ri + e_b \quad (3.2)$$

For non-study state equation 3.1 is rearranged as shown below

$$e_b = v_s - Ri - L\frac{di}{dt} \quad (3.3)$$

According to Newton's second law of motion, the torque of the motor is related as shown below,

$$J\frac{d\omega_m}{dt} = \sum T \quad (3.4)$$

$$T_e = K_f\,\omega_m + J\frac{d\omega_m}{dt} + T_l \quad (3.5)$$

Where,
$K_f$ = Friction constant;
$J$ = Rotor inertia;
$\omega_m$ = Angular velocity;
$T_e$ = Electrical torque;
$T_l$ = Mechanical Load torque;
Where back emf and the electrical torque are

$$e_b = K_e\omega_m;\ T_e = K_t\,\omega_m \quad (3.6)$$

Where,
$K_e$ = Back emf constant;
$K_t$ =Torque constant;
By arranging the Eq.3.3 and 3.4 we get,

$$\frac{di}{dt} = -i\frac{R}{L} - \frac{K_e}{L}\omega_m + \frac{1}{L}v_s \quad (3.7)$$

$$\frac{d\omega_m}{dt} = i\frac{K_t}{J} - \frac{K_f}{J}\omega_m + \frac{1}{J}T_l \quad (3.8)$$

Using Laplace transform to Eq.3.7 and 3.8 we get,

$$\mathcal{L}\left\{\frac{di}{dt} = -i\frac{R}{L} - \frac{K_e}{L} + \frac{1}{L}v_s\right\} \quad (3.9)$$

$$si = -i\frac{R}{L} - \frac{K_e}{L}\omega_m + \frac{1}{L}v_s \quad (3.10)$$

$$\mathcal{L}\left\{\frac{d\omega_m}{dt} = i\frac{K_t}{J} - \frac{K_f}{J}\omega_m + \frac{1}{J}T_L\right\} \quad (3.11)$$

$$s\omega_m = i\frac{K_t}{J} - \frac{K_f}{J}\omega_m + \frac{1}{J}T_l \quad (3.12)$$

At no load $T_l = 0$ therefore,

$$s\omega_m = i\frac{K_t}{J} - \frac{K_f}{J}\omega_m \quad (3.13)$$

$$i = \frac{s\omega_{m} + \frac{K_f}{J}\omega_m}{\frac{K_t}{J}} \quad (3.14)$$

$$\left(\frac{s\omega_{m} + \frac{K_f}{J}\omega_m}{\frac{K_t}{J}}\right)\left(s + \frac{R}{L}\right) = -\frac{K_e}{L}\omega_m + \frac{1}{L}v_s \quad (3.15)$$

Solving above equation we get,

$$v_s = \left\{\frac{s^2JL + sK_fL + sRJ + K_fR + K_eK_t}{K_t}\right\}\omega_m \quad (3.16)$$

The transfer function of the dc motor is the ratio of the angular velocity and the voltage source as follows,

$$G(s) = \frac{\omega_m}{v_s} = \frac{K_t}{s^2JL + s(K_fL + RJ) + K_fR + K_eK_t} \quad (3.17)$$

Assume the friction constant is small i.e. $K_f \rightarrow 0$ , which implies $RJ \gg K_fL$ and $K_eK_t = RK_f$. Therefore equation 3.17 becomes as shown below,

$$G(s) = \frac{\omega_m}{v_s} = \frac{K_t}{s^2JL + sRJ + K_eK_t} \quad (3.18)$$

Rearrange the equation by multiplying numerator and denominator by, $\frac{R}{K_eK_tR}$

We get,

$$G(s) = \frac{\frac{1}{K_e}}{t_mt_es^2 + t_ms + 1} \quad (3.19)$$

Where $t_m$ = mechanical time constant = $\frac{RJ}{K_eK_t}$;

$t_e$ = Electrical time constant = $\frac{L}{R}$;

The mathematical model of BLDC motor is similar to the above derived model of typical DC motor. Except the phases which affects the outcome of the BLDC motor model, particularly resistance and inductance arrangement.

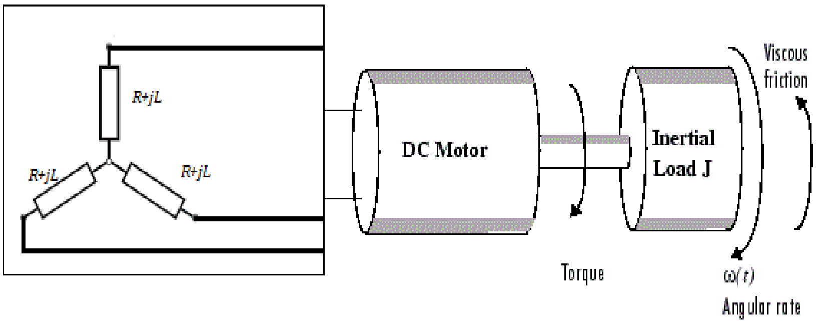


Figure.5. BLDC motor schematic diagram.

The difference in the motor arrangement will affect the time constants, which are very important parameter in the mathematical model. Considering the symmetrical arrangement these time constants will become as shown below,

$$t_m = \sum \frac{RJ}{K_eK_t} = \frac{J\sum R}{K_eK_t} = \frac{J * 3R}{K_eK_t} \quad (3.20)$$

$$t_e = \sum \frac{L}{R} = \frac{L}{\sum R} = \frac{L}{3R} \quad (3.21)$$

Now, considering the phase effect, time constants will become as shown below,

$$t_m = \frac{3R_{ph}J}{(Ke_{L-L}/\sqrt{3})K_t} \quad (3.22)$$

The relation between the time constants by equating their power equations we get,

Ke = Kt x 0.0605

By considering all these effects, the transfer function for the BLDC motor is as follows.

$$G(s) = \frac{\frac{1}{K_e}}{t_mt_es^2 + t_ms + 1}$$

### *3.3 Mathematical modeling for Liquid level control process of two tank system.*

*3.3.1. For two tank interacting system.*

Consider a liquid level system with interaction as shown in the fig [11]. This is a two tank interacting system. In this system the fluid flows through the valves. Assuming the flow of the liquid is laminar. Then, the mass balance equations for both the tanks are taken to derive the transfer function of this interacting system as follows.

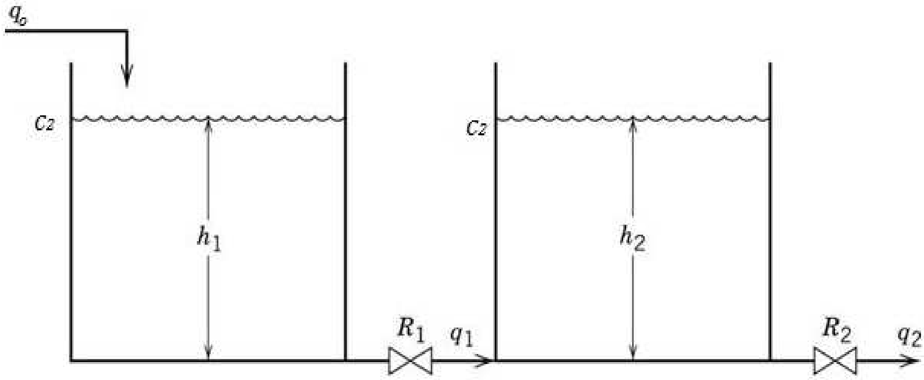


Fig.6 Liquid level two tank interacting process schematic diagram.

For Tank 1:

$$c_1\frac{dh_1}{dt} = q_0 - q_1 \qquad (3.23)$$

$$\frac{h_1 - h_2}{R_1} = q_1 \qquad (3.24)$$

For Tank 2:

$$c_2\frac{dh_2}{dt} = q_1 - q_2 \qquad (3.25)$$

$$\frac{h_2}{R_2} = q_2 \qquad (3.26)$$

Let us take the input variable as inlet flow of tank-1, $q_0$ and output variable as liquid level of the tank-2. Then the Laplace transform of output by Laplace transform of the input gives the transfer function of this model. To get this, let us rearrange the Eq. 3.23 and Eq. 3.25 as follows.

$$q_0 = q_1 + c_1\frac{dh_1}{dt} \qquad (3.27)$$

$$\frac{dh_2}{dt} = \frac{q_1 - q_2}{c_2} \qquad (3.28)$$

Substitute the equations 3.24 and 3.26 in equation 3.27 and 3.28. We get,

$$q_0 = \frac{h_1 - h_2}{R_1} + c_1\frac{dh_1}{dt} \qquad (3.29)$$

$$\frac{dh_2}{dt} = \frac{h_1 - h_2}{R_1 c_2} - \frac{h_2}{R_2 c_2} \qquad (3.30)$$

Taking the Laplace transform on both sides of the Eq. 3.29 and 3.30, we get

$$Q_0(s) = \frac{H_1(s) - H_2(s)}{R_1} + c_1.sH_1(s) \qquad (3.31)$$

$$sH_2(s) = \frac{H_1(s) - H_2(s)}{R_1 c_2} - \frac{H_2(s)}{R_2 c_2} \qquad (3.32)$$

Solving the above equations, in the form of $H_2(s)/Q_0(s)$, this is the transfer function of the liquid level process with interacting two tank system.

$$\frac{H_2(s)}{Q_0(s)} = \frac{R_2}{T_1 T_2 s^2 + s(T_1 + T_2) + R_2 c_1 s + 1}$$

Where $T_1 = R_1 c_1 =$ Time constant of tank-1;
$T_2 = R_2 c_2 =$ Time constant of tank-2;

*3.3.2. For two tank non-interacting system.*

Consider the system as shown in fig. which is a two tank non- interacting system [9]. This system is assumed to be linear and the flow is laminar. If it's not, the system can be linearized by keeping the variation in the variable small. Based on these assumptions, let us obtain the differential equations of the system as follows.

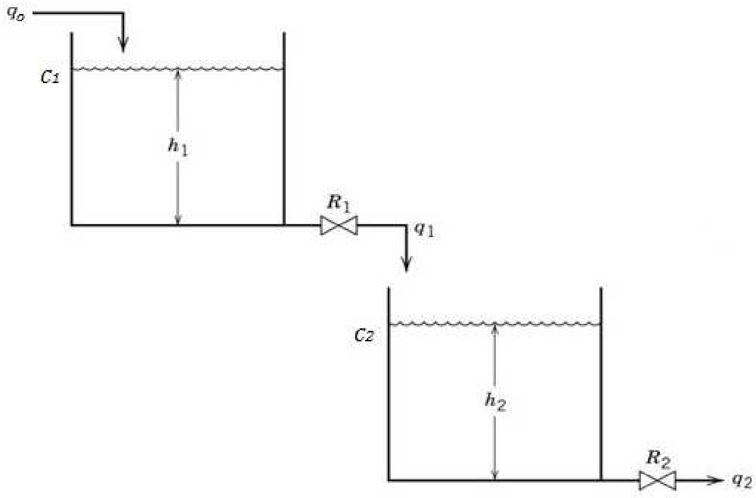


Fig.7 Liquid level two tank non-interacting process schematic diagram.

For Tank 1:

$$c_1 \frac{dh_1}{dt} = q_0 - q_1 \qquad (3.33)$$

$$\frac{h_1}{R_1} = q_1 \qquad (3.34)$$

For Tank 2:

$$c_2 \frac{dh_2}{dt} = q_1 - q_2 \qquad (3.35)$$

$$\frac{h_2}{R_2} = q_2 \qquad (3.36)$$

These equation, which are very similar to the equations of interacting system except Eq. 3.34. Let us take derive the transfer function of this system similar to the interacting system. By solving the above equation taking Laplace transform, we get the transfer function for non-interacting two tank system as follows.

$$\frac{H_2(s)}{Q_0(s)} = \frac{R_2}{T_1 T_2 s^2 + s(T_1 + T_2) + 1}$$

## 4. RESULT AND ANALYSIS

The state and measurement disturbance are assumed to be random disturbance, which is given to the system in terms of percentage error. That is the state disturbance is contiguous by a percentage error of the true states. And the measurement disturbance is contiguous by percentage error of output measurement. Let us discuss the section starting by investigating the effects of the state disturbance, W and measurement disturbance V. on the performance of the state observers and error dynamics of the observer. Then apply the both state and measurement disturbance at same or different percentages, and investigated the error dynamics and observer performance. As the observer is model based method which relays on the actual plant of the system to estimates the states of the system. By adjusting the gain we can correct the error dynamics. Such methods are known as “Gain scheduling”. But here in this analysis we chose to keep the gain constant to demonstrate the effects of the disturbances on the LTI systems.

| State disturbance (w) | Measurement Disturbance (v) |
|---|---|
| 2 | 0 |
| 5 | 0 |
| 10 | 0 |
| 15 | 0 |
| 0 | 2 |
| 0 | 5 |
| 0 | 10 |
| 0 | 15 |

### *4.1 Internal combustion engine.*

The error dynamic of this system is as shown in following fig 8.a-8.e. Where the state disturbance is applied to the system in term of percentage as shown above table. The measurement disturbance is not applied in this first test. Therefore the output measurements of the engine is not affected. Let us assume the manifold intake pressure and engine speed will be scaled by factor of 100. This make us to understand the variations due to the disturbance easily. Right away we can see that the state observer is less sensitive to the disturbance applied by seeing the figures. Especially the throttle angle is more stable than other two states. Speed of the engine will presents more oscillations in the observer error dynamic plots, but these variations in the range and not affect the performance of the system. As the percentage of state disturbance applied to the system increases, we can see there is not much oscillations in the plots. These oscillations are very small, so that it can be easily filtered out. This makes the system less sensitive to the state disturbance.

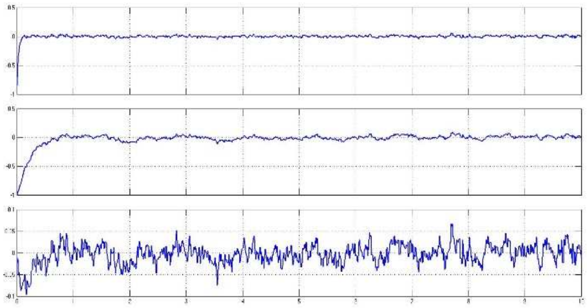

Fig.8.a 2% State disturbance applied to ICE.

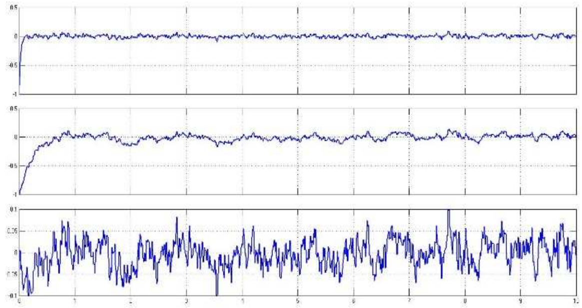

Fig.8.b 5% State disturbance applied to ICE.

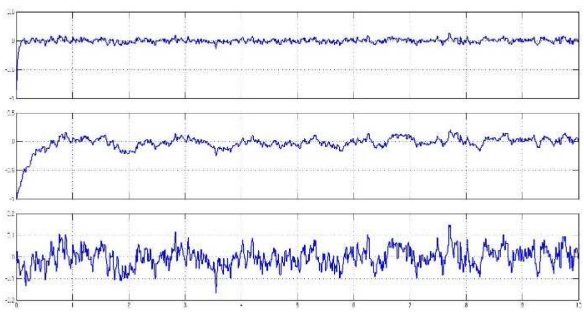

Fig.8.c 10% State disturbance applied to ICE.

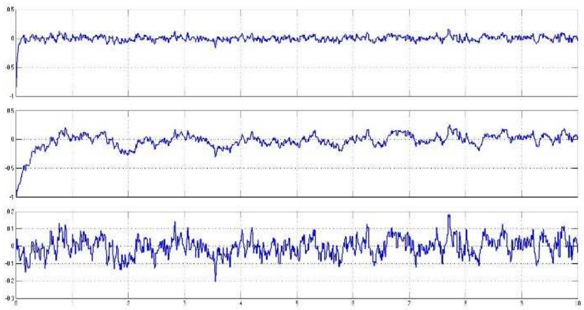

Fig.8.d 15% State disturbance applied to ICE.

In second test, we will not apply state disturbance to the system. Instead we investigate the effects of the measurement disturbance on the system. This measurement disturbance is applied to the output signal. Figure shows the error dynamics when the measurement disturbance on the system was varied from 2%, 3%, 5%, 15% respectively. From these plots we can say, the effect of measurement disturbance is more sensitive than state disturbance. Which results in large variations in the observer error dynamics plot. Here also the throttle angle is more stable than other states of the system. For 5% and more measurement disturbance error dynamics reach the uncontrollable state because of the higher variations.

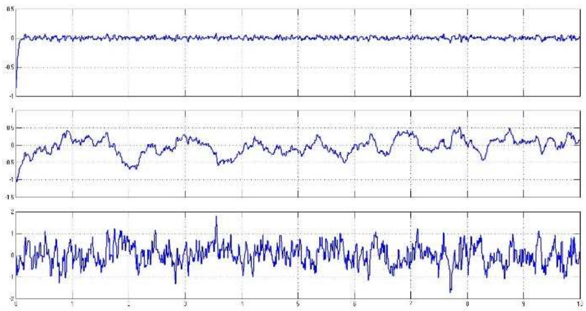

Fig.9.a 2% Measurement disturbance applied to ICE.

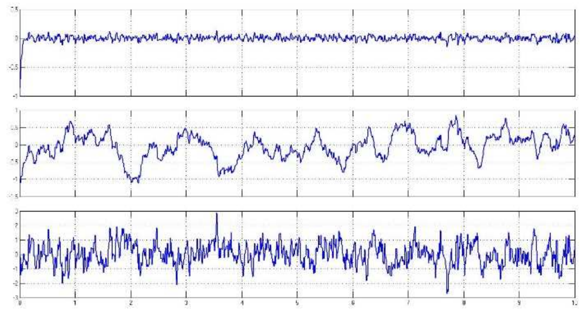

Fig.9.b 5% Measurement disturbance applied to ICE.

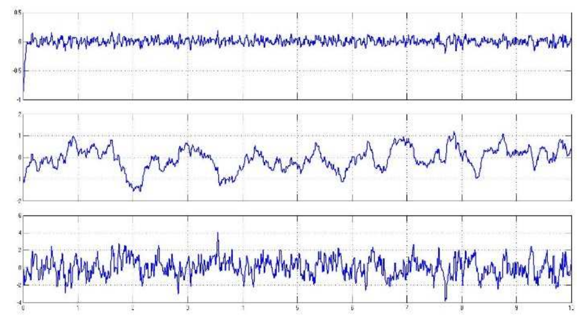

Fig.9.c10% Measurement disturbance applied to ICE.

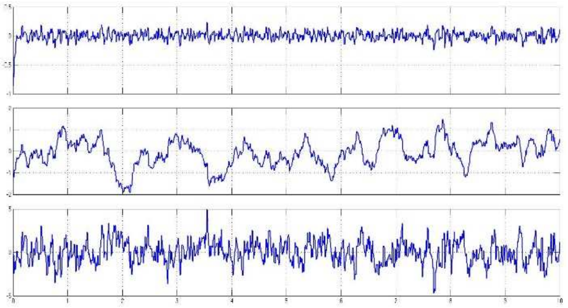

Fig.9.d 15% Measurement disturbance applied to ICE.

The summary of the first and second test results of ICE model are tabulated in the below table.

| State Disturbance(d) in % | Measurement Disturbance(n) in % | Range of observer error | | |
|---|---|---|---|---|
| | | x1 | x2 | X3 |
| 2 | 0 | 0.1 to -0.1 | 0.1 to -0.1 | 0.06 to -0.06 |
| 5 | 0 | 0.1 to -0.1 | 0.2 to -0.2 | 0.1 to -0.1 |
| 10 | 0 | 0.1 to -0.1 | 0.25 to -0.25 | 0.15 to -0.15 |
| 15 | 0 | 0.15 to 0.15 | 0.3 to -0.3 | 0.2 to -0.2 |
| 0 | 2 | 0.1 to -0.1 | 0.6 to -0.6 | 1.8 to -1.8 |
| 0 | 5 | 0.15 to 0.15 | 1 to -1 | 3 to -3 |
| 0 | 10 | 0.2 to -0.2 | 1.5 to -1.5 | 4 to -4 |
| 0 | 15 | 0.25 to -0.25 | -2 to 2 | 5 to-5 |

The final test is, when both the disturbance is applied to the system. We investigated this effects on the system by introducing the disturbances with the percentage pairs as shown in the table given below.

| State disturbance (w) | Measurement dist.(v) |
|---|---|
| 2 | 2 |
| 5 | 5 |
| 10 | 10 |
| 15 | 15 |

Observing the results of these we can see that it only illustrate the state disturbance is less sensitive than measurement disturbance. By further analysis of results when this set of noises applied to the system. When these two disturbances applied it will cross each other to increase or decrease this effect on the state error dynamics.

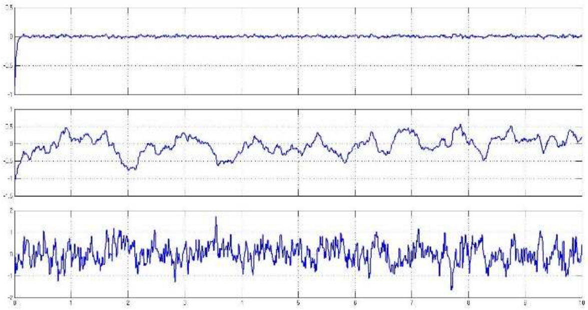

Fig.10.a 2% State disturbance and Measurement disturbance applied to ICE.

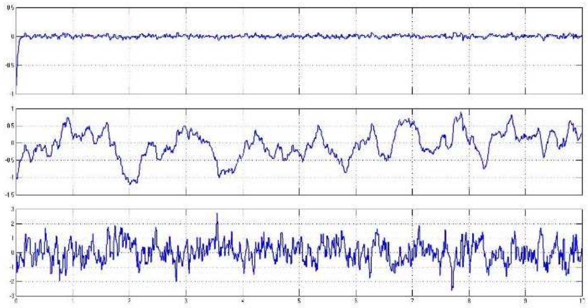

Fig.10.b 5% State disturbance and Measurement disturbance applied to ICE.

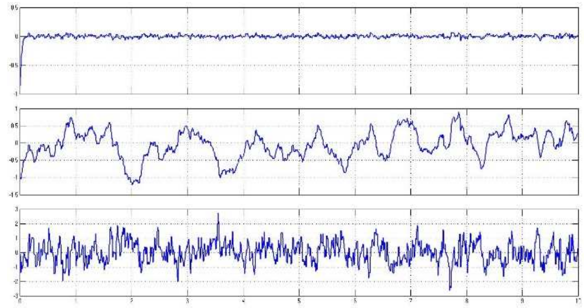

Fig.10.c 10% State disturbance and Measurement disturbance applied to ICE.

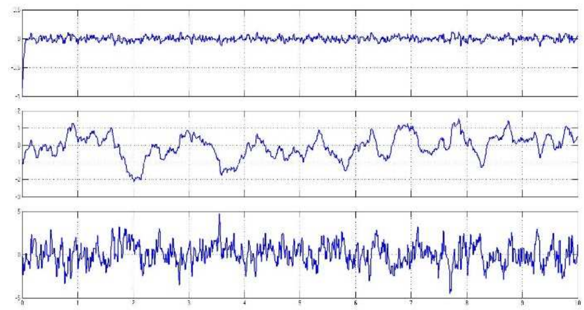

Fig.10.d 15% State disturbance and Measurement disturbance applied to ICE.

### 4.2 ***Brushless DC motor.***

The error dynamic of this system is as shown in following fig 8.a-8.e. The first test on the system is done similar to the tests performed on ICE model. Let us perform the first test on BLDC motor model.

Where the only state disturbance is applied to the system in term of percentage as shown in above table. In this first test, by observing the error dynamics plots we can see that the state observer is more sensitive to the disturbance applied. For 2% disturbance, both the states of the system becomes completely uncontrollable. And makes the system unstable to the state disturbances. Even observer cannot able to maintain the oscillations within the range. As the percentage of state disturbance applied to the system increases, the analysis remain same.

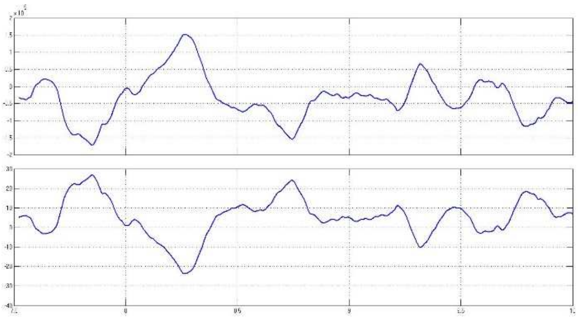

Fig.11.a 2% State disturbance applied to BLDC Motor.

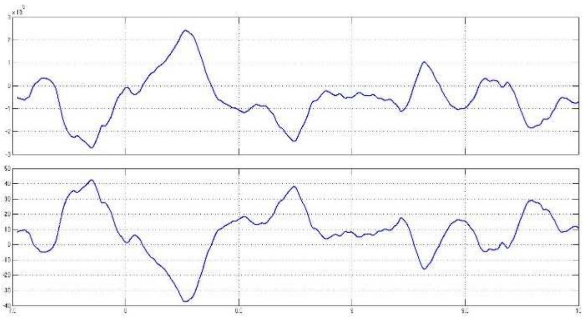

Fig.11.b 5% State disturbance applied to BLDC Motor.

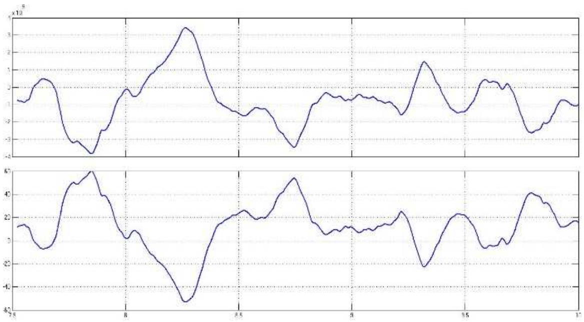

Fig.11.c 10% State disturbance applied to BLDC Motor.

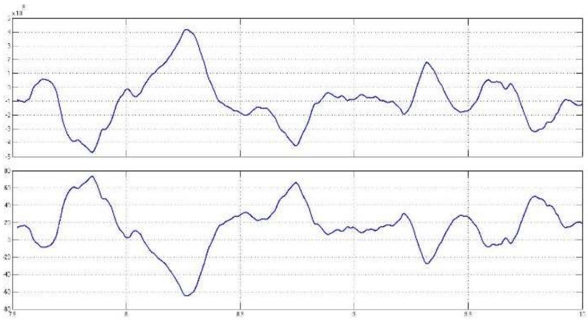

Fig.11.d 15% State disturbance applied to BLDC Motor.

The second test on the system is also done similar to the tests performed on ICE model. We investigate the effects of the measurement disturbance on the BLDC motor model. Fig 12.a-12.b shows the error dynamics when the measurement disturbance on the system. From these plots we can see that the effect of measurement disturbance is less sensitive than state disturbance. Which results in large variations in the observer error dynamics plot. Here both the states (armature voltage and current) of the system will presents lesser range of oscillations, which shows the performance of the observer is high. Even For 5% and more measurement disturbance error dynamics shows small variations.

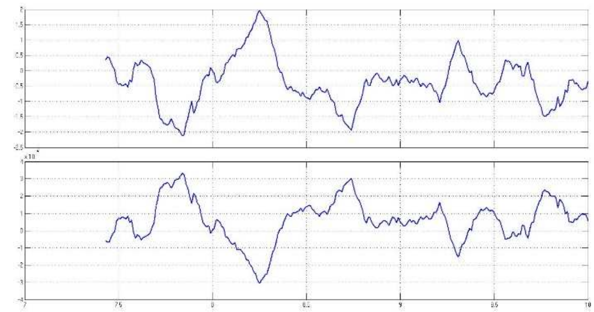

Fig.12.a 2% Measurement disturbance applied to BLDC Motor.

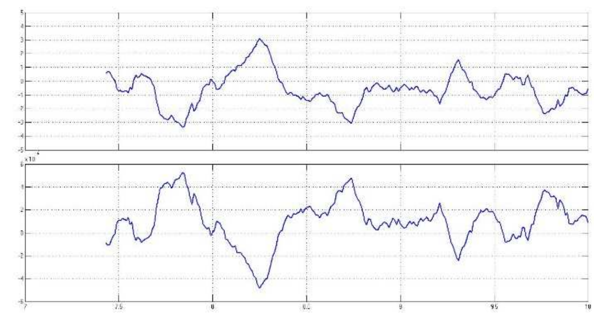

Fig.12.d 5% Measurement disturbance applied to BLDC Motor.

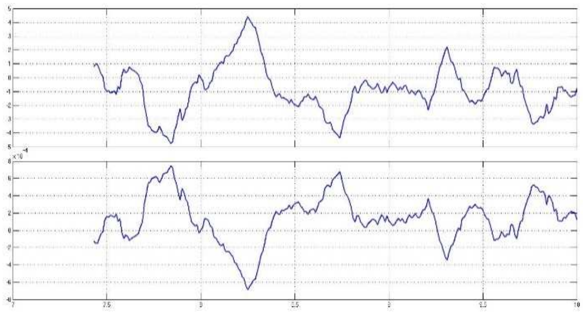

Fig.12.c 10% Measurement disturbance applied to BLDC Motor.

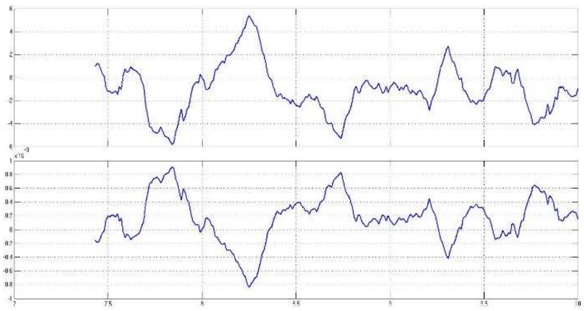

Fig.12.d 15% Measurement disturbance applied to BLDC Motor.

The summary of the first and second test results of BLDC motor model are tabulated in the below table.

| State Disturbance(d) in % | Measurement Disturbance(n) in % | Range of observer error | |
|---|---|---|---|
| | | x1 | x2 |
| 2 | 0 | $\pm 2X10^5$ | 25 to -25 |
| 5 | 0 | $\pm 2.5X10^5$ | 42 to -38 |
| 10 | 0 | $\pm 3.8X10^5$ | 60 to -58 |
| 15 | 0 | $\pm 4.6X10^5$ | 72 to -64 |
| 0 | 2 | 2 to -2 | $\pm 3.2X10^{-4}$ |
| 0 | 5 | 3 to -3 | $\pm 5X10^{-4}$ |
| 0 | 10 | 4.5 to -4.8 | $\pm 7.4X10^{-4}$ |
| 0 | 15 | 5.2 to -5.8 | $\pm 9X10^{-4}$ |

In this final test is, when both the disturbance is applied to the BLDC motor model. We investigated the effects on the system by introducing the both disturbances as shown in the above table. By observing the results of these we can see that it only illustrate the state disturbance is more sensitive than measurement disturbance. By further analysis of results when the percentage increase in disturbance applied to the system. The results shows the similarity in the plots of this and state disturbance.

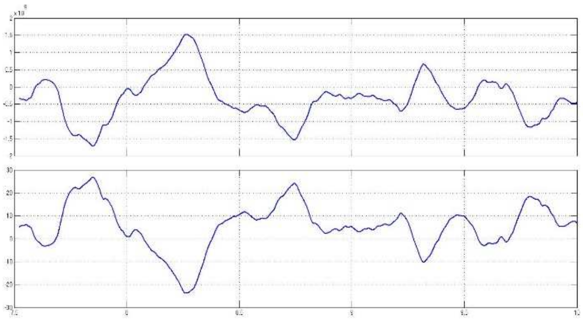

Figure.13.a 2% State and Measurement disturbance applied to BLDC Motor.

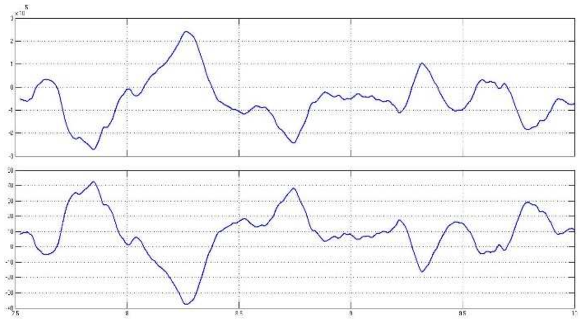

Fig.13.b 5% State and Measurement disturbance applied to BLDC Motor.

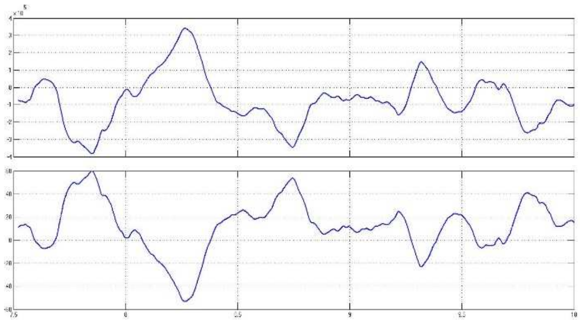

Fig.13.c 10% State and Measurement disturbance applied to BLDC Motor.

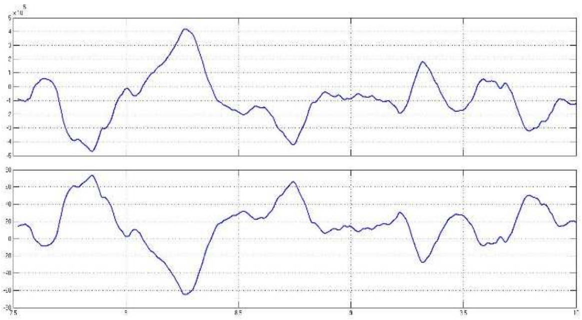

Fig.13.d 15% State and Measurement disturbance applied to BLDC Motor.

***4.3 Liquid level process (two tank interacting and non-interacting system).***

The first test on this system is also similar to the tests performed on ICE model. Where the only state disturbance is applied to the system in term of percentage as shown in above table. By observing the error dynamics plots we can see that the state disturbance doesn't affect much on both interacting and non-interacting liquid level process. Form 2% to 15% disturbance, both the states of the system shows very less variations. This makes the system satisfactorily stable to the state disturbances. The error dynamic of interacting system is shown in fig 14.a-14.d. And for non-interacting system in fig 14.e-14.h as follows.

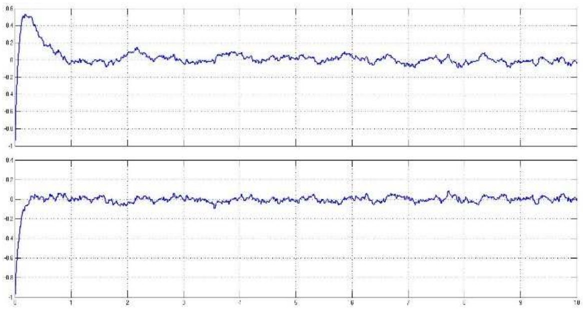

Fig.14.a 2% State disturbance applied to two tank interacting system.

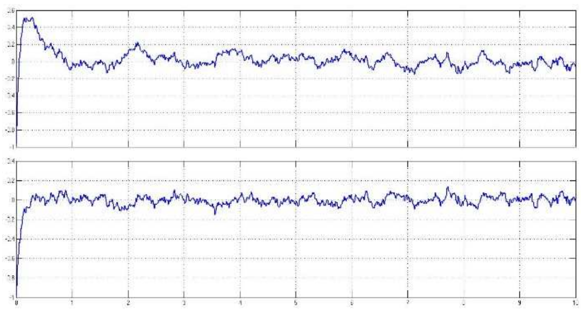

Fig.14.b 5% State disturbance applied to two tank interacting system.

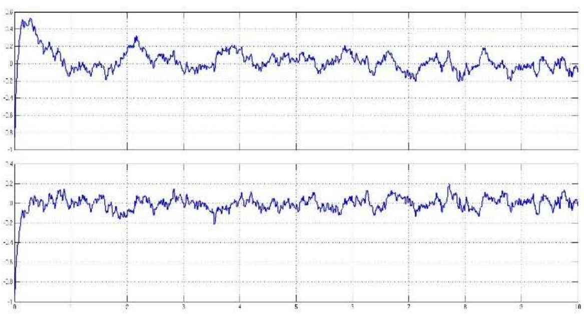

Fig.14.c 10% State disturbance applied to two tank interacting system.

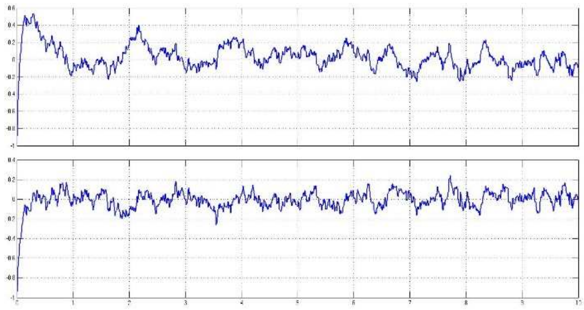

Fig.14.d 15% State disturbance applied to two tank interacting system.

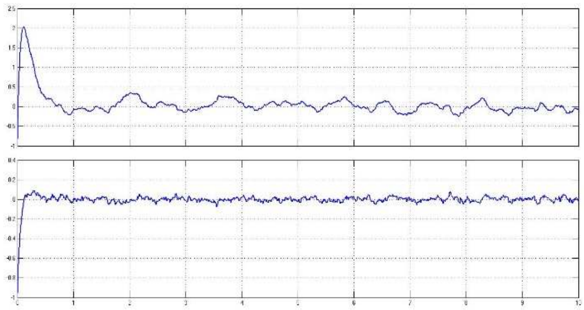

Figure.14.e 2% State disturbance applied to two tank non-interacting system.

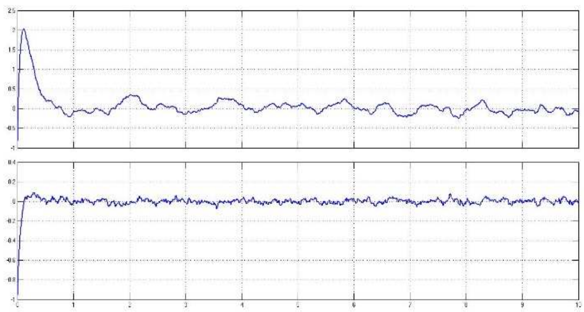

Fig.14.f 5% State disturbance applied to two tank non-interacting system.

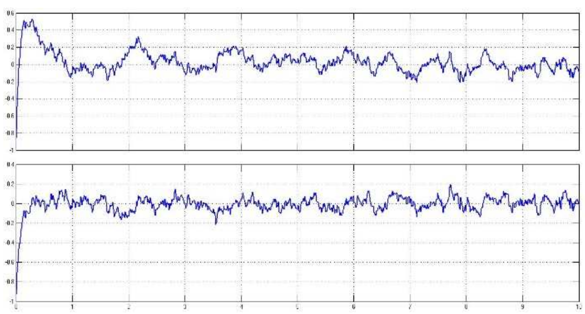

Fig.14.g 10% State disturbance applied to two tank non-interacting system.

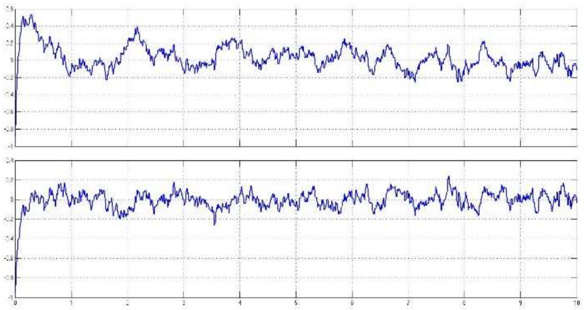

Fig.14.h 15% State disturbance applied to two tank non-interacting system.

In the second test on the system similar to the tests performed on ICE model. We investigate the effects of the measurement disturbance on the Liquid level two tank interacting and non-interacting system. Fig 15.a-12.d shows the error dynamics of interacting system and fig 15.e-15.h for non-interacting system, when the measurement disturbance on the system. From these plots we can see that the effect of measurement disturbance is more sensitive than state disturbance. Which results in large oscillation in the observer error dynamics plot. Here both the states (Level of tank1 and tank2) of the system demonstrate larger range of oscillations, which make the observer states uncontrollable, hence the system is unstable. Even for 2% and more measurement disturbance error dynamics shows large oscillation.

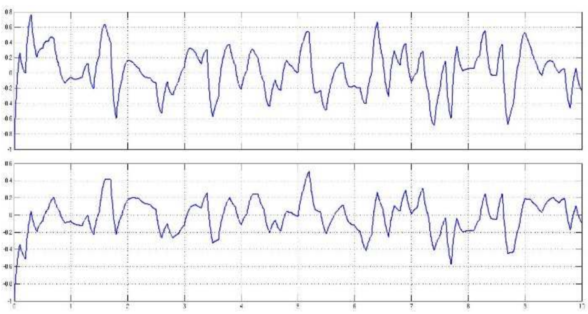

Fig.15.a 2% Measurement disturbance applied to two tank interacting system.

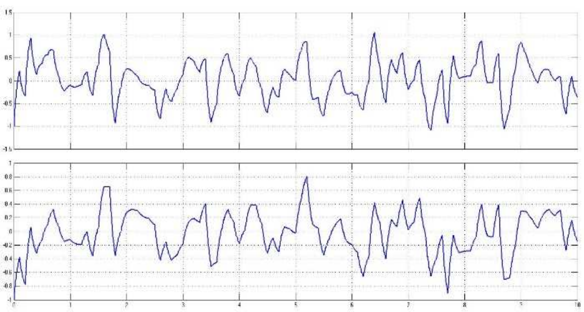

Fig.15.b 5% Measurement disturbance applied to two tank interacting system.

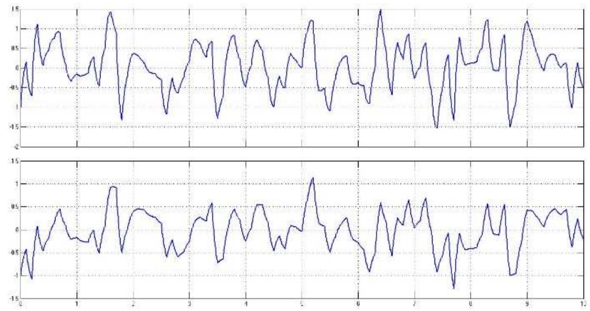

Fig.15.c 10% Measurement disturbance applied to two tank interacting system.

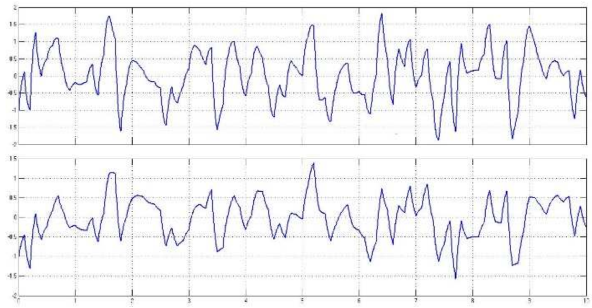

Fig.15.d 15% Measurement disturbance applied to two tank interacting system.

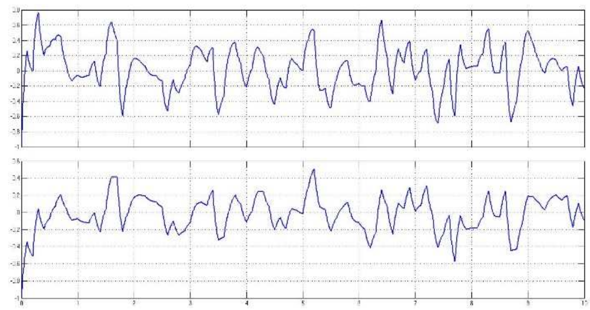

Fig.15.e 2% Measurement disturbance applied to two tank non-interacting system.

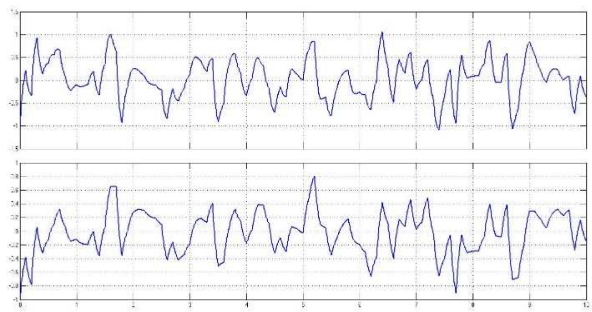

Fig.15.f 5% Measurement disturbance applied to two tank non-interacting system.

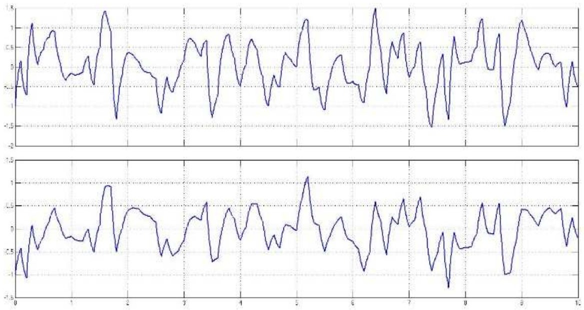

Fig.15.g 10% Measurement disturbance applied to two tank non-interacting system.

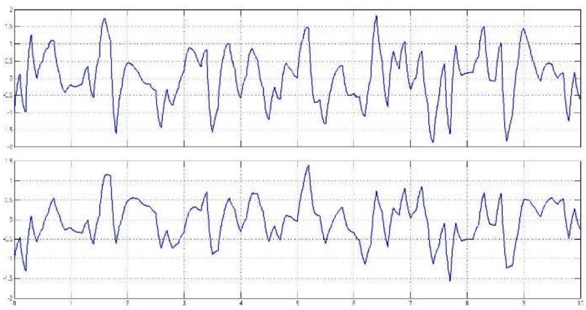

Fig.15.h 15% Measurement disturbance applied to two tank non-interacting system.

The summary of the first and second test results of interacting process and non-interacting Liquid level process were shown approximately same values of range of observer error. Hence the results are tabulated in same table as shown below.

| State Disturbance(d) in % | Measurement Disturbance(n) in % | Range of observer error | |
|---|---|---|---|
| | | x1 | x2 |
| 2 | 0 | 0.16 to -0.16 | 0.1 to -0.1 |
| 5 | 0 | 0.2 to -0.2 | 0.16 to -0.16 |
| 10 | 0 | 0.3 to -0.3 | 0.2 to -0.2 |
| 15 | 0 | 0.4 to -0.4 | 0.24 to -0.24 |
| 0 | 2 | 0.6 to -0.6 | 0.58 to -0.58 |
| 0 | 5 | 1 to -1 | 0.9 to -0.9 |
| 0 | 10 | 1.5 to -1.5 | 1.25 to -1.25 |
| 0 | 15 | 1.8 to -1.8 | 1.6 to -1.6 |

In this final test, when both the disturbance is applied to the Liquid level interacting and non-interacting models. Here we investigated the effects on the system by applying the both disturbances. By analyzing the results of these we can see that it only demonstrates the measurement disturbance is more sensitive than state disturbance. By further analysis the results shows the similarity in the plots of this and state disturbance.

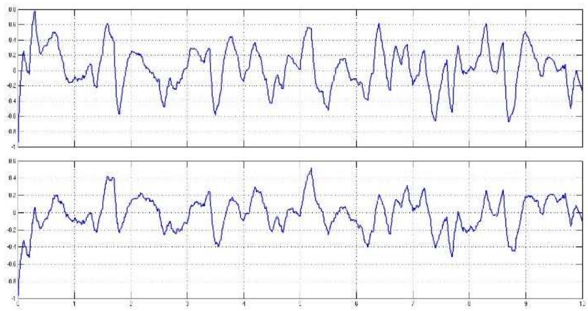

Fig.16.a 2% State and Measurement disturbance applied to two tank interacting system.

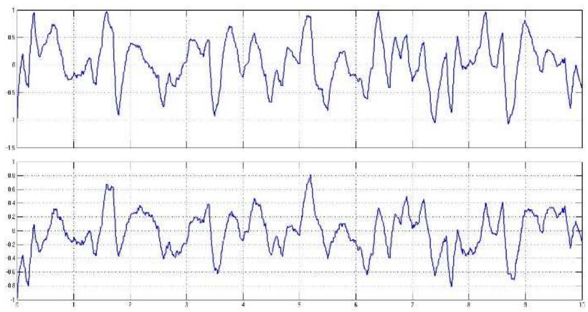

Fig.16.b 5% State and Measurement disturbance applied to two tank interacting system.

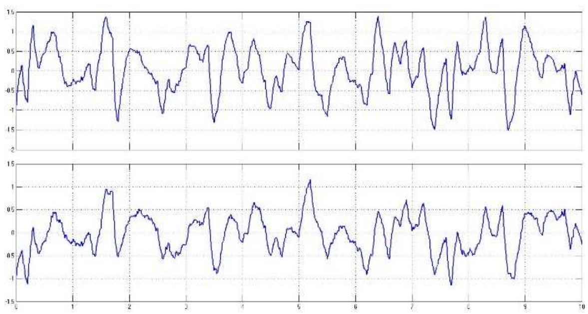

Fig.16.c 10% State and Measurement disturbance applied to two tank interacting system.

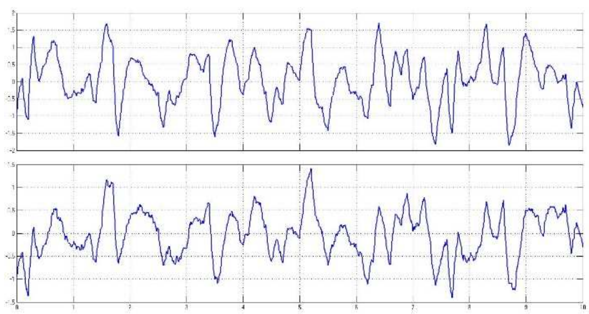

Figure.16.d 15% State and Measurement disturbance applied to two tank interacting system.

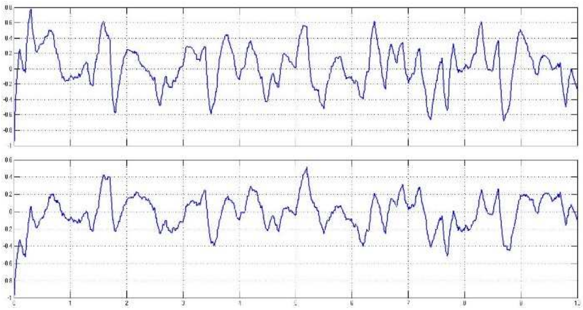

Fig.16.e 2% State and Measurement disturbance applied to two tank non-interacting system.

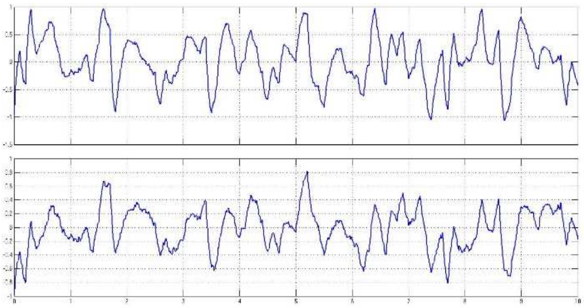

Fig.16.f 5% State and Measurement disturbance applied to two tank non-interacting system.

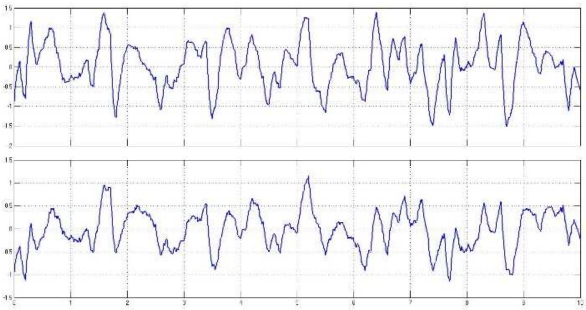

Fig.16.g 10% State and Measurement disturbance applied to two tank non-interacting system.

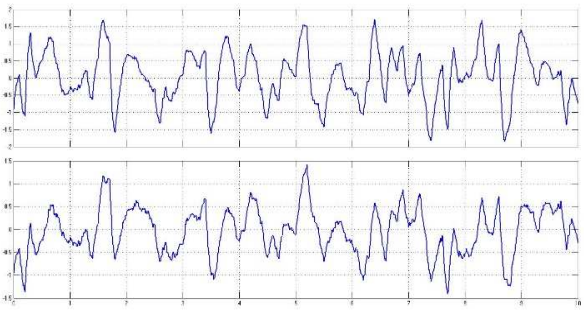

Fig.16.h 15% State and Measurement disturbance applied to two tank non-interacting system.

## 5. CONCLUSION

In this work, we demonstrated that effects of the noises/disturbances on the different linear time invariant systems which work in different domains. As the results shows that the dynamic state observer is very sensitive to the disturbances. Unlike the Kalman filter where the state observer don't consider possible disturbances. But here any disturbance applied will affect the observer directly. And for any system the state observer we presented are great tools for approximation of the states. And they are used practically because controlling a system is not based mainly on observer states, but also on many thing like the sensor measurement, state observer approximation. However, the measurement disturbance can be reduced by the utilization of Low error range, reliable, efficient sensor. And the state observer are often used under state condition where the disturbance will be filtered. With these improvements, the LTI systems state approximations will displays higher performance by keeping the values under range. In this paper we presented how the disturbances will affect the observer states error dynamics of LTI systems. Through simulation in MATLAB, we discovered the effects of state and measurement disturbance, provided with complete analysis of the results.

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